1983 Q2 H.P.

An aircraft flew due east from p to q at u_1 km/h. Wind speed from the south west was v km/h. On the return journey from q to p, due west, the aircraft's speed was u_2 km/h, the wind velocity being unchanged. If the speed of the aircraft in still air was x km/h, $x > v$, show, by resolving along and perpendicular to pq, or otherwise, that

(i) $u_1 - u_2 = v\sqrt{2}$

(ii) $u_1 u_2 = x^2 - v^2$

If $|pq| = d$, find in terms of v, x and d, the time for the two journeys.

EASTward.

$$\vec{V}_{PA} = a\vec{i} + b\vec{j} \quad : \quad a^2+b^2=x^2 ; \quad \vec{V}_{AG} = V\frac{\sqrt{2}}{2}(\vec{i}+\vec{j})$$

$$\vec{V}_{PG} = \vec{V}_{PA} + \vec{V}_{AG} = \left(a+V\frac{\sqrt{2}}{2}\right)\vec{i} + \left(b+V\frac{\sqrt{2}}{2}\right)\vec{j} = u_1\vec{i}$$

WESTward.

$$\left(-a+V\frac{\sqrt{2}}{2}\right)\vec{i} + \left(b+V\frac{\sqrt{2}}{2}\right)\vec{j} = -u_2\vec{i}$$

$$\Rightarrow \quad a + V\frac{\sqrt{2}}{2} = u_1 \quad \text{------} \quad (1)$$

$$-a + V\frac{\sqrt{2}}{2} = -u_2 \quad \text{------} \quad (2)$$

(1) + (2) $\Rightarrow$ $\underline{\underline{u_1 - u_2 = V\sqrt{2}}}$

(1) × −(2) $\quad u_1u_2 = \left(a+V\sqrt{2}/2\right)\left(a-V\frac{\sqrt{2}}{2}\right)$

$$\Rightarrow \quad u_1u_2 = a^2 - \tfrac{1}{2}V^2$$

But $\quad b + V\frac{\sqrt{2}}{2} = 0 \quad \Rightarrow \quad b^2 = -\frac{1}{2}V^2$

$$a^2+b^2 = x^2 \Rightarrow a^2 = x^2 - b^2 = x^2 - \tfrac{1}{2}V^2$$

$$\Rightarrow \quad \underline{\underline{u_1u_2 = x^2 - V^2}}$$

$$\underline{T} = \frac{d}{u_1} + \frac{d}{u_2} = d\left(\frac{1}{u_1}+\frac{1}{u_2}\right) = d\left(\frac{u_1+u_2}{u_1u_2}\right) = \underline{\underline{d\,\frac{\sqrt{4x^2-2V^2}}{(x^2-V^2)}}}$$

As

$$(u_1+u_2)^2 = (u_1-u_2)^2 + 4u_1u_2 = 2V^2 + 4(x^2-V^2) = 4x^2 - 2V^2$$